## TECHNOLOGY, CREATIVITY, IMPLEMENTATION

# DEVELOPMENT OF A MATHEMATICAL MODEL OF VEHICLE ROUTING DURING WINTER ROAD INFRASTRUCTURE MAINTENANCE

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**Summary**

The main goal of the research is to study the problem of vehicle routing during winter maintenance of highways, as well as to build a corresponding mathematical model. It is determined that this problem can be characterized as a typical NP-complete problem. A variety of approaches to solving the problem of routing special equipment are considered. In particular, constructive, composite and mixed (metaheuristic) methods were researched. It was found that some researchers tried to automate the process of route planning, for example, developed application software. It was determined that winter maintenance of roads includes chemical, manual, mechanical and physical snow removal operations. It was established that regardless of the method, the road agency should make management decisions regarding the optimal routes of special vehicle. It is proposed to solve this problem using Graph theory. In this case, the Graph will steal from nodes and lengths between them – routes. The objective function minimizes the number of routes of specialized equipment for winter maintenance of the road network by selecting parameters using binary programming. Since the uncertainty factor is programmed into the proposed mathematical model, we believe that the application of the genetic algorithm for solving the mathematical model will be a promising step in the development of the given research.

**Key words:** winter maintenance, arc route, NP-complete problem, heuristic algorithm, management.

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## 1. Introduction

Until recently, most of the research in the direction of formulating the winter road maintenance model contained simple mathematical solutions and did not pay enough attention to the justification of Service Levels (LOS). Thus, these studies did not take into account the probabilistic nature of factors and characteristics of maintenance of infrastructure facilities in winter. Researchers took this problem very seriously from the point of view of the routing of special vehicle involved in the performance of winter maintenance works *(Perrier, N, Langevin, A, Campbell, JF, 2005)*. The problems of rational placement of operational services and substantiation of material costs were also highlighted separately. The authors emphasized the high importance of vehicle routing for effective winter road maintenance.

In research *(Corberán, Á., 2021; Assad, A., 1995; Eiselt, H., 2000)* the authors suggest the use of various approaches and models of special vehicle routing. For the most part, these routing models are based on the basic tenets of Graph theory. However, it should be noted that the authors do not provide a description of the optimization methods of the proposed models. The works *(Fu, Liping, 2009)* show the use of heuristic methods, which are divided into: constructive, composite and mixed (meta-heuristic) methods. The use of the latter makes it possible to optimize the routing models of special vehicle, which is used to combat winter slippery conditions.

In the work *(Tyupakov, S, 1987)* a quantitative criterion was proposed, which allows to organize uniform sections during winter maintenance of highways according to the degree of urgency of their maintenance. For the first time, the author proposed a consolidated algorithm for calculating indicators of the urgency of road maintenance in the winter period, which was implemented on a computer. The indicator of service urgency is proposed to be determined by the criterion of socio-economic damage from a decrease in the speed of urban road transport, taking into account transport and operating costs. In further research, the author developed and proposed a simulation model of street snow removal by patrol. However, this model does not take into account modern types of technology, increased traffic and mobility on highways.

Thus, the question of justifying the modern mathematical model of vehicle routing for winter maintenance of the highway network, which would allow management decisions to be made, is relevant and necessary. These problems are very complex and specific due to the variety of factors and conditions affecting the performance of works on winter road maintenance. The world experience of solving these problems consists in the application of simulation modeling methods, as a result of which it is possible to save time and costs of material resources of operational organizations that are engaged in winter maintenance of highways.

## 2. The problem of vehicle routing in research

The vehicle routing problem belongs to the arc routing problem (ARP). It should be noted that this complication can be characterized as a typical NP-complete problem with “yes” or “no” decision algorithm answers. As the scale of the problem increases, the number of calculations increases exponentially, so the solution of the routing problem must be based on a heuristic algorithm. Such an algorithm can be solved, for example, using a mixed integer programming model *(Perrier, N, 2005)*. Some researchers propose to solve the problem of optimizing special vehicle routes using: arc routing model with time windows *(Hagani, A., 2001)*, nonlinear algorithm *(Eglese, H., 1994)*, “tabu search” algorithm *(Haghani, A., 2001)*, Graph theory *(Xie, B., Li , Y., & Jin, L., 2013)*.

The use of heuristic procedures was proposed for the routing of vehicles involved in winter maintenance *(Fu, Liping, 2009)*. They can be broadly divided into three categories: constructive methods; composite methods; adaptation of metaheuristics.

Constructive vehicle routing methods can be divided into four classes: sequential route construction methods, parallel route construction methods, cluster first – route later, and optimization methods based on the basic principles of Graph theory.

Composite methods are used to take into account restrictions in the model on the number and power of vehicles, consumption of materials, time of work, etc. This group of methods is based on heuristic approaches.

Metaheuristic methods are based on the simulation approach and are used to solve the problem of the randomness of model variables and the unpredictability of factors that affect the model of winter road maintenance. Metaheuristics methods allow simulation of various events and situations.

Some researchers tried to automate the process of route planning, for example, a simulation model of street snow removal patrol *(Tyupakov, S., 1987)*, GeoRoute application software *(Campbell, J., 2000)*. But research on route optimization of de-icing vehicles, salt distribution is still rare.

Thus, the purpose of the research is to develop a mathematical model of vehicle routing for winter maintenance of the highway network.

## 3. Development of a mathematical model of vehicle routing

The main purpose of road maintenance works in winter conditions is to eliminate winter slippage on the surface within the prescribed time limits, to ensure the passage of motor vehicles, and to ensure the necessary adhesion of tires to the surface in places where traffic is difficult. Winter operational maintenance of roads includes specific operations related to the seasonal problem – the fight against winter slippery conditions. These operations include the distribution of chemical reagents and abrasive materials (a chemical method of snow removal); manual, mechanical, physical snow removal (table 1). The chemical method includes the use of chemical, fractional or combined materials. The manual method includes human labor with simple tools. The mechanical method is the use of machinery with specialized equipment. One of the key elements in the mechanical maintenance of roads in winter is snow removal vehicles, which includes sand throwers, loading equipment, tractors, motor graders, rotary snow plows, salt spreaders, bulldozers, as well as attached equipment. In different countries of the world, depending on the climatic zones, the equipment may differ in its constructive qualities. An example of a physical method with snow is the installation of heating elements in road elements.

Table 1

**Characteristics of different methods of snow removal [adapted from *(Xie, B., Li, Y., & Jin, L., 2013)*]**

| Characteristic | Snow removal method | | | |
|---|---|---|---|---|
| | Chemical (use of anti-icing materials) | Manual | Mechanical | Physical |
| Efficiency | High | Low | High | High |
| Energy consumption | Low | High | High | Relatively high |
| Impact of transport | Very low | Heavy | Relatively heavy | Zero |
| The cost of the object | Low | Low | The purchase price is high | The cost of construction is high |
| Cost of use | Relatively high | High | Relatively high | Relatively high |
| Impact on the environment | Heavy | Low | Damage to the coating | No pollution, no damage |

All the considered methods require the road agency to make decisions about the optimal routes of special vehicle and the costs of anti-icing materials.

The presence of snow deposits on the carriageway of roads, even in a small amount, leads to a decrease in speed, road safety and driving comfort. This problem is particularly significant in urban conditions, since the intensity of traffic in populated areas is much higher than in intercity traffic.

The problem of justifying the strategy of winter maintenance of roads should be discussed from the standpoint of the system "environment – road conditions – traffic flows – road agency".

In *(Tyupakov, S., 1987)* the objective function was proposed as a criterion for comparing winter road maintenance strategies:

$$F = \sum_{j=1}\sum_{i=1} C_i Z_{ij} + \sum_{j=1}\sum_{m=1} C_m \theta_{mj} + \sum_{m=1} C^n{}_m P_m \rightarrow mi, \tag{1}$$

where $S$ – is the amount of snowfall before operation;

$K$ – number of road sections;

$M$ – the number of mechanized brigades;

$Z_{ij}$ – the time during which the transport has losses due to a decrease in the speed of traffic on the $i$-th road in $j$ – the event of snowfall;

$\theta_{mj}$ – crew working time;

$P_m$ – idle time;

C – cost.

Since the values $S$, $Z$, $\theta$, $P$ have a random character, the function $F$ is also a random grand, accordingly, it is necessary to use the mathematical expectation for its characterization:

$$F' = M(F). \tag{2}$$

The problem of routing special equipment for winter maintenance of roads is directly related to service levels, as thanks to the solution of this problem, it is possible to determine the optimal number of necessary resources (units of special equipment) to perform the work.

Since promising methods for solving this problem are the use of heuristic algorithms, we consider it appropriate to stop at the first stage of using graph theory to solve the problem of vehicle routing. In this case, the graph of the problem will look like this:

$$G = \{V, L(v_i, v_j)\} \rightarrow \left| \begin{matrix} V \in \{v_0 : v_n\} \\ L(v_i, v_j) > 0 \end{matrix} \right., \tag{3}$$

where $V$ – a set of nodes, from the initial one $v_0$ to the final $v_n$;

$L(v_i, v_j)$ – an arc established between two nodes.

To optimize this problem, it is necessary to present the graph in the form of a mathematical model:

$$f = \sum_{k=1}^{k=K} \sum_{i=0}^{i=n-1} \sum_{j=1}^{j=n} l_{ij} x_{ijk} + \varepsilon \rightarrow min, \tag{4}$$

where $f$ – the aim function;

$l_{ij}$ – arc length between nodes, $l_{ij} \in [v_i, v_j]$;

$x_{ijk}$ – the number of routes between nodes, $x_{ijk} \in [v_i, v_j]$;

*K* – the number of paths (passages) of special vehicle;

*ε* – uncertainty factor.

Limitations are suggested to the model *(Xie, B., Li, Y., & Jin, L., 2013)*:

1) the area of performance of works is served by an agency whose facilities are located in this area;

2) metrological conditions and natural features of the service area are known in advance;

3) each vehicle serves only one district;

4) the road is serviced at least once.

As a result of simulation, the decision regarding the route is made by binary programming (NP-complete problem) under the condition:

$$x_{ijk} \in \begin{cases} 1 \to L(v_i, v_j) \in k \\ 0 \to otherwise \end{cases}. \quad (5)$$

In addition, it is worth taking into account uncertainty and random events, which are usually indicated in the standards in the form of a percentage deviation from the obtained value.

The results of the research have practical value and can be used to make management decisions regarding winter road maintenance in conditions of financial and resource uncertainty, including those related to force majeure circumstances. Because route optimization has a long-term economic and social effect.

## 4. Conclusions

The problem of routing special vehicle during winter road maintenance is a typical NP-complete problem, so it is difficult to find an optimal solution with an exact algorithm in a reasonable calculation time. In particular, in modern scientific literature, it is proposed to solve the problem using a genetic algorithm. Since the genetic algorithm is one of the tools of intelligent search, its application allows not to directly influence the variables. However, it should be based on the use of a set of parameters for coding, which the developed mathematical model can serve.

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