MODERNIZATION OF REAL ESTATE VALUATION METHODOLOGY BY COMPARATIVE METHOD

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Summary
This study is devoted to the issue of determining the price of a real estate object based on the use of statistical market evaluations of similar objects. In the course of the research, a technique for determining the price of a real estate object was developed using parametric equations of the least squares method, which allows using an unlimited amount of input information and obtaining equalized values of price-forming factors. In contrast to the existing methods of the comparative approach to determining the price of the object of assessment, the proposed method allows you to obtain the value of the price-forming factors, and then the price of the object without establishing the difference between the factors influencing the price, as is the case with the classical approach of processing the input database. Approbation of the developed methodology showed the effectiveness of the possibility of its application in practice.

Key words: price, real estate, assessment, factors. technique, method of least squares.

DOI https://doi.org/10.23856/6323

1. Introduction

Three main methodological approaches are used in the evaluation activity: income, expenditure and comparative, the theory and practice of which is in constant dynamic development and undergoes certain modernization (Büyükkaraciğan N., 2021; Pagourtzi, E., et, all, 2003). In modern conditions, real estate valuation methods based on the application of mathematical modeling methods are becoming more and more important (IC. Yeh, TK. Hsu, 2018). In particular, it is proposed to use combinations of multicriteria methods (Koziol-Kaczko D., 2014; Taillardier, F., & Abi-Zeid, I., 2013), as well as regression models (Hnennyi O., 2012), while evaluation procedures are performed using GIS tools – technologies (Bozdağ, A., Ertunç, E., 2020; Mete, M. and Yomralioglu, T., 2021). The use of the latest evaluation methods allows to a certain extent to eliminate a number of shortcomings that are inherent in analytical evaluation methods, when a number of evaluation indicators are the subjective judgment of individual evaluators. However, it should be noted that the reliability of the results of assessment by mathematical modeling methods depends to a large extent on the reliability of the arguments of the input data.
One of the methods widely used in evaluation activities is the method of comparing the prices of similar sales or the comparative method. The essence of this method is to determine the price of the appraised object based on the price of already sold similar objects, taking into account the difference in the influence of each factor on the appraised value of the appraised objects. In most cases, experts use an analytical method in which, based on the expert judgments of an expert appraiser, the main price-forming factors for a given type of real estate are determined and appropriate corrections are made to these factors for differences between the appraised object and the object of comparison.

To solve this problem, a system of linear equations is created, in which the differences between the evaluation results of similar objects and the evaluated one are expressed through corrections to the main factors selected by expert evaluators. At the same time, the number of selected analogue objects should not exceed the number of price-forming factors (Dekhtyarenko Yu.F., Mantsevich Yu.M., Palekha Yu.M., 2013; Drapikovskyi O.I., Ivanova I.B., Krumelis Yu.V., 2015). Thus, the basis of this method is the difference between the factors of the object of assessment and the analogue. Note that differences in professionalism, moral and ethical preferences of expert evaluators often lead to inconsistencies in the evaluation procedure and raise a number of questions.

In this case, the system of equations has a single unambiguous solution. Hence, when compiling a system of linear equations of corrections of analogue objects, such a number of analogue objects is selected that would meet the set requirements for the unambiguity of obtaining final results, which ultimately leads to an unreasonable sampling of the remaining large array of input data. Let's consider the possibility of solving the given problem for the case when the number of analogue objects exceeds the number of evaluation factors (indicators) and the price of the evaluation object is a function not of the differences in the price-forming factors, but of the arguments of the factors themselves.

2. The main part

In this publication, we will consider the theoretical and practical aspects of using the apparatus of the method of least squares to determine the estimated value of the object of assessment regardless of the number of similar objects, which allows to use a wide statistical spectrum of the influence of various factors on the final assessment result.

Formulation of the problem. To develop a methodology for determining the price of the appraised object based on the prices of realized analogue objects, using the parametric least squares method.

The purpose of this study is to develop an innovative approach to real estate valuation based on statistical analysis of the real estate market, which would provide appraisers with a theoretically grounded mathematical mechanism for real estate valuation regardless of the number of similar sales.

3. Materials and research Methods

Let’s represent the price of the evaluation object with an expression

\[ a_0 x_1 + b_0 x_2 + \ldots + g_0 x_k = L_0, \]  
where
а₀, b₀, …, g₀ – empirical coefficients for price-forming factors, determined by an expert-appraiser on the basis of existing legal acts;

x₁, x₂, …, xₖ – correction coefficients determined on the basis of mathematical processing of the array of input data (sale of similar objects);

L₀ – the price of the evaluated object.

Thus, the solution to the given problem consists in determining the most reliable values of the correction coefficients x.

Let the number of n analog objects be selected in the process of evaluation work, and the number of price-forming factors is equal to the number k.

In this case, it is necessary to solve the system of equations

In the following equations:

\[ \alpha_1 x_1 + b_1 x_2 + \ldots + g_1 x_k = l_1; \]
\[ \alpha_2 x_1 + b_2 x_2 + \ldots + g_2 x_k = l_2; \]
\[ \ldots \]
\[ \alpha_k x_1 + b_k x_2 + \ldots + g_k x_k = l_n \]

x₁, x₂, …, xₖ – correction coefficients of factors that affect the price of the object of assessment;

l₁, l₂, …, lₙ – sale prices of similar objects;

α₁, …, αₖ; b₁, …, bₖ; g₁, …, gₖ – respectively, the coefficients for the factors influencing the price of the object of evaluation, determined by the expert.

Let the number of similar objects be selected in the process of evaluation, the number of which is equal to the number of price-forming factors (n=k).

Then, the system of equations (2) has a single solution for determining the unknowns x₁, x₂, …, xₖ. Having found the value of the specified unknowns and substituting them into formula (1), it becomes possible to determine the price of the evaluated real estate object. Thus, the task is solved. However, the solution of this variant of the problem does not provide an opportunity to assess the accuracy of the obtained results.

In the practice of appraisal works, a situation arises when it is possible to select the number of analogue objects, which significantly exceeds the number of significant price-forming factors, i.e. n>k. In this case, the system of equations (2) is indeterminate and a set of solutions is possible. For each individual decision, the values of the unknowns will depend both on the values of the prices of similar objects and on the values of the normalized coefficients. Among the set of value systems for unknowns, the one with the smallest sum of squares of the corrections is considered the best, that is, when \( \sum v_i^2 = \text{min} \).

Let us consider the theoretical aspects of this approach. We will present the results of the evaluation of analogue objects in the form of functions

\[ f_i(t_1 + x_1, t_2 + x_2, \ldots, t_k + x_k) = l_i = v_i, \]  

where

\( t_1, t_2, \ldots, t_k \) – values of price-forming factors;

\( v_i \) – difference (correction) between the value of the price of the i-th analog object calculated as a result of the problem solution and its known value.

After expansion into the Taylor series of functions (3), limiting ourselves to terms of the first degree of accuracy, we have:

\[ f_i(t_1, t_2, \ldots, t_k) + a_i x_1 + b_i x_2 + \ldots + g_i x_k - l_i = v_i \]

In the given expression, \( a_i, b_i, \ldots, g_i \) are partial derivatives of function (3) with respect to variables.
Assume: \( f_i (t_1, t_2, \ldots, t_k) = 0 \). Then, equation (4) will take the form:

\[
a_i x_i + b_i x_i + \ldots + g_i x_i - l_i = v_i.
\]

(5)

Given formula (5), let’s write down the system of linear correction equations in an expanded form, in which \( n \) is the number of objects of comparison, and \( k \) is a set of indicators that have a significant impact on the price of real estate of this type. We have:

\[
a_1 x_1 + b_1 x_2 + \ldots + g_1 x_k - l_1 = V_1; \\
a_2 x_1 + b_2 x_2 + \ldots + g_2 x_k - l_2 = V_2; \\
a_k x_1 + b_k x_2 + \ldots + g_k x_k - l_n = V_n.
\]

(6)

From the system of equations of corrections go to the system of normal equations.

As a result, a system of normal equations is obtained, in which the number of equations is equal to the number of unknowns.

The resulting system of equations has the form:

\[

c_{ij} x_i + c_{ij} x_j + \ldots + c_{ik} x_k = 0
\]

(7)

The solution of the system of normal equations is performed by well-known methods, which are widely covered in modern literature. As a result of the solution of the system of equations, the values \( x_i \) of the price-forming factors are obtained, and then it becomes possible to calculate, according to formulas (6), corrections in the analog objects accepted for mathematical processing.

According to Bessel’s formula:

\[
m = \sqrt{\frac{vv}{n-1}}
\]

(8)

it becomes possible to assess the accuracy of determining the price of an individual object.

The mean squared error calculated by the formula characterizes the conditions for obtaining the prices of similar objects.

We will formulate a general technological scheme for determining the price of real estate objects using the proposed method:

– analysis of the real estate market and the formation of a database by statistical sampling of similar objects and the main price-forming factors of this type of real estate;
– determination by expert or experimental studies of the numerical values of the influence of individual price-forming factors of similar objects and the evaluated object;
– compilation of linear equations of corrections taking into account the values of coefficients during factor corrections;
– transition from the system of linear correction equations to the system of normal equations;
– solving the system of normal equations and finding the values of the price-forming factors;
– determination of corrections in the prices of similar objects;
– assessment of the accuracy of the obtained results. The determination of the mean square error of the price of a separate analog object and the object of assessment is determined by formula (1) of the price of the assessed object with taking into account the obtained values of \( x_i \).
4. Results

Let’s consider the methodology of practical implementation of the proposed method.
1. In the process of analyzing the real estate market, five evaluated similar objects are selected, which in terms of their price-forming factors are close to the evaluation conditions of the object of evaluation. At the same time, three factors have been identified that have a dominant influence on the pricing of real estate of this type.
2. Let’s create a database of input data.
The unit price of similar objects is:
\[ l_1 = 50; \quad l_2 = 45; \quad l_3 = 48; \quad l_4 = 52; \quad l_5 = 46 \text{ (unit)} \]

On a five-point scale, the numerical values of the conditions of influence on the price of similar objects are determined:
\[ a_1 = 4; \quad a_2 = 2; \quad a_3 = 1; \quad a_4 = 4; \quad a_5 = 4; \quad b_1 = 2; \quad b_2 = 3; \quad b_3 = 2; \quad b_4 = 1; \quad b_5 = -1; \quad c_1 = 1; \quad c_2 = 2; \quad c_3 = 2; \quad c_4 = 1; \quad c_5 = -1 \]
and the evaluation object \( a_0 = 3, \quad b_0 = 1, \quad c_0 = 1 \).
3. We compile a system of linear equations according to formulas (2).
We get:
\[ 4x_1 + 2x_2 + x_k = 50; \]
\[ 2x_1 + 3x_2 + 2x_k = 45; \]
\[ x_1 + 2x_2 + 2x_k = 48; \]
\[ 3x_1 + x_2 + x_k = 52; \]
\[ 4x_1 - x_2 - x_k = 46 \]

4. We find the price of the object of assessment.
4.1. From the solution of the system of the first three equations, we get:
\[ x_1 = 11.60; \quad x_2 = -14.60; \quad x_3 = 32.80 \]
Using formula (1), we determine the price of the object of evaluation:
\[ L_0 = 3 \times 11.60 + 1 \times (-14.60) + 32.80 = 53 \text{ (unit)} \]

4.2. When using all analog objects from the system of five linear equations, we move to the system of normal equations using formulas (7). We get the following system of equations:
\[ 46x_1 + 15x_2 + 9x_k = 678; \]
\[ 15x_1 + 19x_2 + 14x_k = 33; \]
\[ 9x_1 + 14x_2 + 11x_k = 242 \]

From the solution of the given system of equations, we get:
\[ x_1 = 14.25; \quad x_2 = -17.57; \quad x_3 = 32.88 \]
Taking into account the obtained values, the price of the object of evaluation is
\[ L_0 = 3 \times 14.25 + 1 \times (-17.57) + 32.88 = 58 \text{ (unit)} \]

5. Assessment of the accuracy of the obtained results.
According to the formulas of the system of equations (6), we calculate the corrections to previously evaluated objects – analogs, and with formula (8) – the mean squared error of the assessment of a separate object.
We have: $v_1 = 4.7$, $v_2 = -3.5$, $v_3 = -3.1$, $v_4 = 6.1$, $v_5 = -4.2$

Control: $\sum v^+ = \sum -v = |10.8|$. Hence $\sum vv = 98.8$ and $m = \sqrt{98.8/4} = 4.96$

Thus, the average squared error of determining the price of an analog object does not exceed 10 percent, which indicates a proper sampling of analogs and the coefficients of influence of individual factors determined for them.

5. Conclusions

As a result of the conducted research, a method of real estate valuation using the method of least squares was developed.

The proposed method and the developed technological scheme for processing input data, in contrast to existing methods, allow you to directly determine the resulting values of price-forming factors and the price of the evaluated object regardless of the number of similar objects, which to a certain extent reduces the influence of subjective factors on the final evaluation result.

The practical implementation and technological scheme of the proposed methodology will facilitate their application by assessment practitioners.

References