PEDAGOGICAL SCIENCES

THEORETICAL FOUNDATIONS FOR THE DEVELOPMENT OF HIGH SCHOOL STUDENTS' MATHEMATICAL ABILITIES IN STUDYING ALGEBRA AND PRE-CALCULUS

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Summary

The article provides a theoretical analysis of psychological, pedagogical, and scientific-methodical literature in the context of the development of high school students' mathematical abilities in studying algebra and pre-calculus. The purpose of the provided research is to determine the theoretical foundations for the development of high school students' mathematical abilities in the process of studying algebra and pre-calculus. In accordance with the purpose, the most important psychological and pedagogical conditions are disclosed in the research work, contributing to a deep, quick and easy mastery of math knowledge and skills, as well as ensuring the development of individual psychological characteristics of high school students – their mathematical abilities. The scientific foundations for the implementation of the problem-based approach to the development of high school students' mathematical abilities are described, the theoretical aspects of its implementation in the process of studying algebra and pre-calculus are determined. The content of the zones of proximal mathematical development is determined; their structure in studying algebra and pre-calculus by high school students is defined, the connection with the educational material content and components of mathematical abilities is revealed. To achieve the purpose, the following research methods were used: a theoretical analysis (problems of a problem-based approach to the development of mathematical abilities of high school students); a structured system analysis (for the problem-based system build-up), a contensive-theoretical generalization and design (in the formulation of conclusions and definition of subject matter for further researches).

Keywords: mathematical abilities of high school students, high school students, algebra and pre-calculus, training and learning activities, zones of proximal development, problem-based approach.

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1. Introduction

A modern renewal of the purpose and education subject matter is closely related to the problem of the personality's creative development. An important factor in the development of such a personality is the formation of students' skills to apply the gained knowledge in real-life situations, as well as the formation of students' interest in mathematics and the development of

their mathematical abilities. Therefore, one of the main tasks of the school is the organization of effective learning activities that will ensure the development of the student's abilities and his formation as a creative personality.

The problem of personality development in the school-age period was developed by such psychologists as P. Ya. Galperina, Z. I. Kalmykova, V. A. Krutetskyi, O. M. Leontieva, N. O. Menchynska, V. O. Moliako, J. Piaget, S. L. Rubinstein, B. M. Teplov, and others. A significant contribution to the study of students' mathematical abilities was made by such educationalists as Sh. Amonashvili, D. Dewey, V. Levites, V. A. Sukhomlynskyi, and others, as well as methodologists A. K. Artemova, V. A. Gusev, A. M. Kolmogorov, D. Pólya, S. P. Semenets, Z. I. Slepkan, J. S. Chashechnikova, and others. But, so far, the issue of development of specifically high school students' mathematical abilities in the process of studying algebra and pre-calculus, in particular the problem of creating of psychological and pedagogical conditions in the educational process for the development of these individual psychological features, specifics of the organization of mathematics learning activities, remain unexplored.

The purpose of the article: to determine the theoretical foundations for the development of high school students' mathematical abilities in studying algebra and pre-calculus.

To achieve the purpose, the following tasks are being solved: to analyze psychological and pedagogical as well as training and studying literature on the raised issue; to determine the subject matter, structure, general psychological and pedagogical conditions for the development of mathematical abilities of high school students; to reveal the features of the organization of high school students' mathematics learning activities in the process of studying algebra and pre-calculus.

2. Subject matter and structure of mathematical abilities

A significant contribution to the solution of this problem was made by V. A. Krutetskyi. By definition of the researcher, mathematical abilities are the individual psychological characteristics that meet the requirements of mathematics learning activities and determine the success of creative mastery of mathematics as a school subject, in particular a quick, easy mastery of knowledge and skills in the field of mathematics *(Krutetskyi, 1968: 98)*.

The components of mathematical abilities include: the ability to formalize mathematical material, separate form from content, abstract from specific quantitative relations and spatial forms, and operate with formal structures of relations and connections; the ability to generalize mathematical material, to identify the essentials (ignoring the insignificant), while cognizing the general in a different by form; the ability to operate with numerical and sign symbols; the ability to consistent, correctly dissected logical thinking associated with the need for proofs, justification, conclusions; the ability to shorten the process of thinking, to think in collapsed outlines; the ability to reverse the process of thinking (transition from direct to reverse thought); flexibility of thinking, the ability to switch from one mental operation to another; mathematical memory (memory for generalizations, formalized structures, logical schemes); spatial awareness *(Krutetsky, 1968: 104)*.

In this study we adhere to the opinion that in the basis of mathematical abilities is the structure, designed by S. P. Semenets and L. M. Semenets *(Semenets, 2013)*:

a system-forming component (mathematical orientation of the mind as a personal characteristic manifested in structural and mathematical thinking, interest in the models' build-up, research and implementation);

a coding-formalized component: (ability to formalize in the process of establishing the mathematical structure of theoretical and practical material, the creation and study of sign-symbolic interpretations (models) of problem-based situations);

a cognitive-generalizing component (the ability to generalize the content of mathematical material on several levels, finding alternative (variable) and rational solutions, mental (intuitive) "capture" of the formal structure (algorithm) based on a particular case)

a mnemonic-generalizing component (memorization of mathematical material on different levels of theoretical generalization: memory for typical relations (formulas), general schemes of thinking (algorithms), the structure of methods and ways for solving problems (proving and researching).

The student's interest and inclination to mathematics inspire him to study the subject intensively, which in turn ensures the development of mathematical abilities. Developed mathematical abilities and work provide new achievements and successes in the field of mathematics that cause student satisfaction. As a result, there is an even greater interest in mathematics, and there appears a desire to study it even more.

Besides abilities, interests, and inclinations, certain character traits also contribute to achieving success in the activity. The researchers distinguish the following personal qualities: concentration and determination, diligence and efficiency, stubbornness, and individual style. Often, even the outstanding abilities of the individual do not provide significant achievements *(Semenets, 2015: 98)*.

The idea, that talented and, generally, gifted students get everything in an easy, simple way, without any particular difficulties, is unconvincing. Persistent learning, long and hard work are essential for the development of abilities. As a rule, abilities are always combined with exceptional diligence. No wonder the talented people emphasize that talent is the work, multiplied by patience, it is a tendency to endless work.

The development of the student is a process of his self-creation as a personality in the activity. Types of activities include learning, which acts as a form of obtaining theoretical knowledge in the process of solving training problems. The structure of holistic human activity is formed by needs, motives, goals, conditions, and means of achieving goals, actions, and operations *(Semenets, 2015: 108–109)*.

3. Psychological and pedagogical conditions for the development of senior school students' mathematical abilities

Considering the polystructurality and multifunctionality of the studied phenomenon, the development of mathematical abilities, in our opinion, depends on certain (defined) psychological and pedagogical conditions, which include: biological inheritance; the activity performed by the student; the social environment in which the student finds himself.

The biological inheritance is the transfer of the relevant qualities and features from parents to children *(Zaichenko, 2016: 40)*. The researchers believe that the inherited natural (biological) characteristics of a person, that is, inclinations – innate anatomical and physiological characteristics of the nervous system, are a natural basis for the development of human abilities *(Pavelkiv, 2009: 34)*. They may or may not be realized, depending on whether opportunities were provided for this. If, for example, a child is born in a family of healthy parents, where all possible conditions for his development are created, then, of course, the relevant inclinations, that in due time will be transformed into abilities, will develop better than with children born in a dysfunctional family. The psychologists believe that inclinations do not completely determine

the development of abilities. Abilities can be developed only under certain conditions of students' life and activities.

We share the opinion of S. P. Semenets that the development of a student is the process of his self-creation as a personality in the activities *(Semenets, 2015: 109)*. At the senior school age, the choice of a profession becomes an important factor in the development of personality, establishing a close connection between professional and educational interests. In this regard, learning activities are subject to specific goals for achieving success in the future. Therefore, an important psychological factor in the education of high school students is the motivation to study.

Active and successful implementation of mathematics learning activities depends on positive motivation, namely, on educational and cognitive interest in studying mathematics. The stability of cognitive interest is based on internal motives, determined by the content and significance of the work, its subjective value.

An important condition for the development of mathematical abilities with high school students is the formation of the operational component of the activity. This requires from the teacher the systematic formation of students' mental actions and methods of mental activity, training them in methods of proving and solving problems, the formation of methods of educational work, in particular, methods of self-control, correction, and self-assessment of mathematics learning activities. It is worth noting, that any type of students' cognitive activities is based on such general mental operations as analysis and synthesis.

An important condition for the development of mathematical abilities is the formation in high school students of such mental actions as abstracting and generalizing. Abstracting is a mental action aimed at identifying in objects and phenomena of the essential and separating the nonessential. The result of abstracting, as a rule, is abstraction – images created by the human mind *(Slepkan, 2006: 41)*. Here it is important to teach students to separate the significant from the insignificant, the general from the individual, logically go from abstract (general) to specific (partial). Strictly speaking, for the development of students' mathematical abilities, it is important to introduce a deductive way of knowing the truth, which adequately corresponds to the content of mathematics, its deductive essence.

Generalizing is the process of identifying the common in objects and phenomena. It is an integral part of various types of mathematics learning activities: formulation of concepts, solving of math problems, proving of theorems, and research of functions. The designated contensive and theoretical action is pivotal in the cognitive-generalizing and mnemonic-generalizing components of mathematical abilities.

Taking into account the specific character of the studied phenomenon, we believe that a deep understanding of mathematics is impossible without the development of mathematical intuition. It is intuition that ensures the mental "capture" of the formal structure (algorithm) on the basis of one partial case. Due to mathematical intuition, an instantaneous intellectual comprehension of a solution "is coming" for a mathematical task (a math problem), which is further supported by logic. For its development, "qualitative" questions are useful, which presuppose verbal solutions and require non-standard considerations, hypothesis formulation, as well as the informal analysis. A manifestation of mathematical intuition, as one of the components of mathematical abilities, is a sudden "enlightenment" – "insight". As noted by V. A. Krutetskyi, the basis of "insight" is the ability to generalize and the ability to think in collapsed structures *(Krutetskyi, 1968: 338)*. It is noteworthy that cases of such "enlightenment", the sudden finding of a correct and original idea is more common with high school students. Therefore, by developing mathematical intuition, stimulating "wow-experience" in

high school students, conditions are created for the development of their personal complex, which is mathematical abilities.

To psychological and pedagogical conditions for the development of high school students' mathematical abilities we include the organization of mathematics learning activities in accordance with the psychological principles of developmental learning, substantiated by Z. I. Kalmykova: the principle of problematicity, individualization and differentiation, the harmonious development of various components of thinking, the formation of algorithmic and heuristic methods of mental activity, the special organization of mnemonic activity *(Kalmykova, 1979)*. We believe that these principles are fundamental in the process of creation a methodological system for the development of high school students' mathematical abilities, its content, procedural and control and evaluative components.

The next condition that affects the development of personality, its individual psychological characteristics, is the social environment. The social environment is composed of people, their relationships, things, tools, linguistic means, spiritual values, created by them. The period of early adolescence is characterized by increased anxiety, which is associated with self-esteem. High school students change attitudes toward studying, which is associated with an internally motivated need for self-determination. The attitude of the high school student to the teacher is also undergoing changes, the relationship with the teacher becomes more complex and differentiated. In the image of the ideal teacher, his individual human qualities come to the fore – the ability to understand, cordiality, empathy, assertiveness. The second place is occupied by professional competence, and in the third place is the ability to manage fairly. For high school students, the relationship between teacher and students is built purely on the basis of mutual understanding and respect for each other *(Sergeenkova and et al., 2012: 60)*.

Success in studying, the level of high school students' aspirations in mathematics learning activities, and hence the development of their mathematical abilities is due to the psychological atmosphere, especially the positive psychological climate. An important role in this is played by the positive emotional component, interpersonal (subject-subject) relationships, and adequate self-esteem of high school students. It has been experimentally proven that students with low self-esteem have a decrease in interest in studying, there is a lack of confidence in their strengths, mental capabilities.

4. Zones of proximal mathematical development

It is worth noting that the development of individual psychological qualities of high school students personality depends on the activity process of cooperation with the teacher and peers, during which the zones of the proximal mathematical development are created: the measure of independence is established, expedient mathematics learning activities are organized, the process of interiorization is provided. Here the nonlinear organization of training in algebra and the pre-calculus should be embodied, the problem-based approach to the development of mathematics learning activities should be realized, and levels of contensive and theoretical generalization of problems should correlate with zones of proximal mathematical development of high school students. *Under zones of proximal mathematical development, we understand* such component of studying mathematics in which, first, in the result of joint activities the measure of the student independence in mastering the way of actions in the course of solving a new type of math problems is defined, second, an expedient collective (collectively distributed) mathematics learning activities for mastering new knowledge and skills, development of his personal qualities is organized, third, in the course of such studying mathematics its phenomenological

characteristic becomes interiorization, as a result of which a certain type of math problem is solved by the student independently, and his personal qualities have a higher level of development *(Semenets et al., 2019: 84)*.

We associate the creation of zones of proximal mathematical development of students with the planning and organization of mathematics learning activities in accordance with the principle of developmental continuity, according to which each subsequent type of problem differs from the previous by the higher level of contensive and theoretical generalization. Proceeding from the fact that the level of contensive and theoretical generalization of the problem-based system of training in mathematics correlates with the zone of proximal mathematical development of subjects of mathematics learning activities *(Semenets, 2015: 134)*, in training high school students in algebra and pre-calculus we distinguish four zones of proximal development: basic, training, training-theoretical and training-exploratory.

Level I: **basic zone** – basic (applied) problems in algebra and pre-calculus are formulated and solved; abilities are formed to create mathematical models, to establish methods of action in the process of solving particular problems in algebra and pre-calculus, to plan them, control their implementation and assess the level of mastery.

Level II: **training zone** – educational problems in algebra and pre-calculus are formulated and solved, abilities are formed to create educational models, to establish methods of action in the process of solving typical problems in algebra and pre-calculus, to plan them, to perform self-control and self-correction, to make self-assessment of achievement.

Level III: **training-theoretical zone** – training and theoretical math problems in algebra and pre-calculus are formulated and solved, skills are formed to create training theoretical models, to establish and apply methods for solving problems of contensive lines of algebra and pre-calculus, general logical and general mathematical methods of solution (proving and exploration), as well as the ability to perform self-control and self-correction, to make self-assessment of achievement.

Level IV: **training-exploratory zone** – training and exploratory problems in algebra and pre-calculus are formulated and solved, math exploratory abilities are formed, as well as the ability to perform a theoretical analysis of training and mathematical scientific literature, to apply methods of mathematical perception and exploration, to determine the contensive components of scientific exploration (object, subject, goal, tasks, hypothesis, scientific novelty, scientific and mathematical methodology).

5. A problem-based approach

The subject matter of training high school students in algebra and pre-calculus contains two large blocks: theoretical material and math problems. The introduction of a competency-based approach implies that math problems should play a fundamental and systemic role. Here, one of the conceptual provisions is the statement that the mathematics learning activities of high school students are presented in the form of a holistic problem-based system. Theoretical analysis of the problem raised led to the conclusion that the problem-based approach is understood as a learning activity, based on a problem-based structure, a component of which is a training math problem, which, on the one hand, is aimed at an external object, and on the other hand contains implicitly expressed requirements to the subject who solves it *(Lovyanova et al., 2010)*.

While agreeing with this definition, we need to note that any activity has problem-based structure, and therefore a problem-based approach, in our opinion, represents a unity of

universal ways of planning, organization, development, and diagnostics of the subject's activities, in which external manifestations (means of action in the process of solving math problems, oral and written speech, the answer to the question posed) are combined with its internal manifestations (needs, motives, values, memory, thinking, self-control, self-esteem, and abilities) on the system level. Taking into consideration this formulation, *the problem-based approach in math training* is a conjunction of methods for planning, organizing, developing and diagnosing the mathematics learning activities of a subject, which on the system level combines its external manifestations (means of action in the process of solving math problems, mathematical modeling, oral and written mathematical speech, the formulation of the answer) with internal manifestations (motives for studying mathematics, values-based attitude to it, mathematical memory, mathematical thinking, self-control and self-assessment of mastering the math material, mathematical abilities).

By implementing a problem-based approach to training high school students in algebra and pre-calculus, we distinguish the following three types of problems:

– math problems;
– math problems in the structure of mathematics learning activities;
– competency-based math problems.

Under *math problems*, we mean problems, the statement, and requirement of which concern mathematical objects and which can be solved by all mathematical means *(Tarasenkova et al., 2015)*.

The well-known classification basis of math problems is the content of the statement in their structure. Therefore, four types of math problems are classified in the math learning theory: on the calculation (it is necessary to find a number or a set of numbers), on plotting (math problems on plotting graphs, figures, etc.), on proving (it is necessary to prove the formulated statement) and math problems on exploration (it is necessary to explore something) *(Slepkan, 2006)*. According to the current mathematics curriculum for students of 10–11 grades of secondary schools, the main contensive lines are: functional; equations and inequalities; elements of combinatorics; elements of probability theory, and elements of mathematical statistics. A significant place is occupied by math problems on calculating and simplifying transcendental expressions, as well as problems involving identity substitution of trigonometric, power, and logarithmic expressions.

In the presented study, the fundamental position is that abilities, including mathematical ones, are formed and developed in the process of specially organized activity, in the structure of which positive (internal) motivation and interest play a key role. We agree that such a developmental triplet should be realized in math training: interest in mathematics ⇔ mathematics learning activities ⇔ mathematical abilities. We support the idea that mathematics learning activities have a problem-based structure, which is constructed in accordance with the principle of developmental continuity. Therefore, mathematics training should be organized in such a way that each subsequent type of math problem differs from the previous one by a higher level of contensive-theoretical generalization. The realization of such an idea made it possible to single out basic, training, training-theoretical, and training-exploratory math problems *(Semenets, 2015)*.

In our opinion, the study of key topics in algebra and pre-calculus should begin with the formulation and solution of *basic math problems*. Here we are talking about applied problems, according to the results of the solution of which mathematical models are created, the output relations are singled out, which are realized in many partial cases. Such problems are classified as math problems of the first level of contensive-theoretical generalization, they are

the foundation for the introduction (formulation) of a new mathematical concept and solving math problems.

The second level of contensive-theoretical generalization is occupied by *training math problems* in algebra and pre-calculus. The result of solving training math problems is the creation of a generalized way of action (training model), which is implemented in the process of solving typical math problems. Therefore, a training math problem in algebra and pre-calculus presupposes generalization of typical math problems of the course, constructing a way (method) for solving them, in which the leading role is played by training activities on self-control and self-assessment.

On the third level of contensive and theoretical generalization of the problem-based system, we single out *training-theoretical math problems* in algebra and pre-calculus. Here generalized methods of action are formed during studying contensive lines, mastering by high school students of general mathematical methods (mathematical modeling, vector, coordinate, mathematical induction, boundaries, differential, and integral calculus), as well as special methods of algebra (factorization, substitution, interval, equivalent transformation, graphical, functional and others).

The fourth level of the problem-based system is occupied by *training-exploratory math problems* in algebra and pre-calculus, which, in addition to the contensive-theoretical generalization level, differ in the degree of the obtained product novelty. By solving such math problems, students perform exploratory steps, gain new knowledge, and form objectively new ways of activity. Training-exploratory problems in algebra and pre-calculus are usually solved by mathematically gifted high school students in the course of mathematical competitions, in particular, competitions for the defense of student works at the Small Academy of Sciences.

In this way, the problem-based system of training in algebra and pre-calculus is fully consistent with one of the key phenomenological characteristics of mathematical abilities – the ability to generalize the content of mathematics. The mathematics learning activities, organized in the form of a problem-based system, actualizes all structural components of mathematical abilities.

Another type of math problem that we single out is *competency-based problems* in algebra and pre-calculus. The researcher M.V. Dubova considers competency-based math problems as a form of educational material organization, modeled in the form of a quasi-life situation, which is designed to form disciplinary, interdisciplinary and key competencies of students *(Dubova, 2011)*.

We share the opinion of O. G. Kuzminska and N. V. Morze that competency-based math problems contribute to intellectual activity and independence both in the process of solving and assessment (self-assessment, mutual assessment) of an intellectual task, they contribute to the ability for goal-setting, assessment, effective action and reflection *(Kuzminska et al., 2008)*. Such tasks are the condition, means, and at the same time the result of the formation of mathematical competencies of high school students. It should be mentioned here that the effectiveness of this process is due to the level of mathematical abilities development, and on the other hand, the solution of competence problems in algebra and pre-calculus contributes to the development of high school students' mathematical abilities. In our opinion, the place of such math problems in the presented problem-based system of training in algebra and pre-calculus should be determined in accordance with the principle of developmental continuity and correlated with basic, training, training-theoretical and training-exploratory math problems. In the process of solving them, external manifestations of competence (ways of actions in the process of solving problems, oral and written speech, alternative solutions, answering the question posed) and its

internal manifestations (motives, values, memory, thinking, self-esteem and mathematical abilities should combine in a consistent manner).

This system of math problems of different types will promote the digestion of both the theoretical part of the training material of mathematics and the practical one, will contribute to the development of logical and creative thinking, development of mathematical modeling skills will form mathematics learning activities according to the logic of ascent from abstract (general) to specific (partial). This principle is fundamental in constructing the concept of a model of mathematics learning activities.

We share the opinion that mathematics learning activities consist of five structural components *(Semenets, 2015: 116)*:

– need-motivational component (based on the need for personal self-affirmation, professional self-determination in the process of studying mathematics);

– design component (provides for the design of a four-level problem-based system of mathematics learning in accordance with the principle of developmental continuity);

– constructive component (reveals the content and structure of actions and operations in the process of solving mathematical problems, building up symbolic models);

– implementation component (provides step-by-step execution of actions and operations in accordance with the created training model);

– reflected component (presupposes self-analysis, self-assessment, self-control of mathematics learning activities).

6. Conclusions

Thus, the development of the mathematical abilities of high school students during training in algebra and pre-calculus presupposes the holistic observance of psychological and pedagogical conditions: biological inheritance; the activity performed by the student; the social environment in which the student finds himself. The construction of the process of training in mathematics should be organized in the form of training and mathematical activities in compliance with the principle of developmental continuity, in the structure of which we single out basic, training, training-theoretical and training-exploratory math problems, which correspond to four zones of proximal mathematical development.

Our further research work will be aimed at developing a methodological system that will contribute to the development of high school students' mathematical abilities at algebra and pre-calculus lessons.

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